# EXPERIMENTAL INVESTIGATION OF FREE CONVECTION ABOVE A HEATED HORIZONTAL WIRE

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Results are presented of an experimental investigation of the convective plume above a fine horizontal wire, heated by a constant current in air and in water. The temperature distribution in the plume was investigated using the IAB-451 shadow instrument in the diffraction interferometry method. The experimental results are in good agreement with laminar convection theories above a linear heat source. In the air, a comparison was made with the experimental results of other authors.

A theoretical investigation of free laminar convection from a linear heat source was made in [1-4], of which the most detailed are [3, 4]. Almost simultaneously with [4], two experimental investigations were published, dealing with the two-dimensional stationary laminar convective plume in air [5, 6]. Reference [5] investigated the velocity and temperature fields for a single thermal condition. The velocity field was investigated by photographing a dust-laden stream in stroboscopic light. The temperature field was measured from interferograms obtained on a Mach–Zender interferometer. In [6] a detailed investigation was made of the temperature field in a two-dimensional convective plume. The temperature measurements were made by means of a thermocouple probe traversed in the plume region. The results of these investigations showed good argeement with the numerical solution of the two-dimensional problem [3] for air (Prandtl number $P = 0.7$). No experimental investigation have been made, apparently, of convection from a linear heat source in a capillary liquid.

1. In this paper the heat source was a platinum wire of diameter 0.095 mm and length 9.13 cm, stretched at a height H above the surface of a horizontal flat plate of size 28 × 10 × cm, and parallel to the shorter side. One end of the wire was fastened by means of a bronze spring, which applied the required tension in the wire and avoided its becoming slack when heated. The platinum wire, which also acted as a resistance thermometer, formed one element of a balanced bridge circuit. The current in the bridge during operation was measured in terms of the voltage drop over a standard 1-Ω resistor. The circuit was supplied with constant current from a large capacity battery system. All the conducting wires in the low-resistance branches of the bridge circuit were calibrated beforehand, their resistances was taken into account in calculating the resistance of the platinum wire.

The model was located inside a rectangular insulated chamber of dimensions 60 × 30 cm, and height 60 cm. The end walls, of size 60 × 60 cm, contained protected plane-parallel windows of the IAB-451 equipment. The parallelism of the windows was checked by an autocollimation method. The insulated chamber was located in the working space of the IAB-451 shadow instrument so that the end viewing windows were strictly perpendicular, and the heated wire was strictly parallel to the optical axis of the instrument.

The structure of convective flow above a thin heated cylinder is such that the diffraction interferometry method [8, 9] is well suited for studying the plume temperature field. To apply the method a vertical split of width 0.01 mm was mounted in the focal plane of the IAB-451 collimator, and a wire of 0.04 mm was located at the focus of the viewing tube. The light source was a DRSh-250 lamp with a filter which passed the yellow line of the mercury spectrum, of wave length $\lambda = 579$ mμ.

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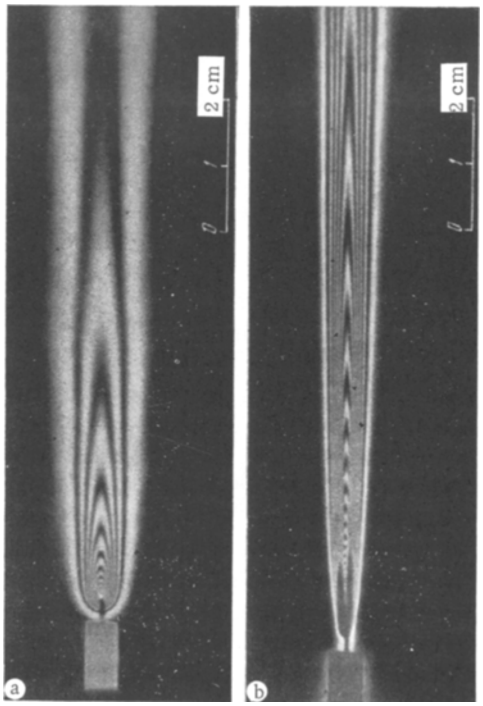

Fig. 1

It was established by special preliminary tests that the proximity of the plate (the thermal conductivity of the plate material is $\varkappa = 6.10^{-4}$ cal/cm sec°C) affects the convective heat transfer from the wire, both in air and in water, only when the wire is in the immediate vinicity of the plate. For $H \geq 3$, the heat transfer from the wire is given by the conventional empirical relation for free convection from thin cylinders in an infinite region [7], with Rayleigh numbers in the range $10^{-2}$ to 1:

$$N = 1.03\ \mathrm{R}^{1/9} \tag{1.1}$$

where the Nusselt and Rayleigh numbers are calculated on wire diameter. This means that the entire thermal power dissipated by the wire goes to form convective flow above the source. In the experiments investigating the plume, the distance H was such that Eq. (1.1) was satisfied.

Photographs of the observed interference picture are shown in Fig. 1 (*a* is the convective plume in air, per unit thermal power dissipated by the heated wire $q_l = 45.3 \cdot 10^{-3}$ cal/cm sec; b is for distilled water, $q_l = 85.6 \cdot 10^{-3}$ cal/cm sec). The photographs (particularly Fig. 1a) show a periodic variation of the intensity and width of the interference fringes. This effect, due to the existence in the plume of a vertical component of temperature gradient, does not complicate calculation of the interference picture however. The interference fringes in the photographs are isotherms in the plane perpendicular to the heat source passing through its center. An end effect is observed above the ends of the wire. The plume contracts somewhat on top in a vertical plane passing through the source. The end effect was taken into account in reducing the interferograms. The degree of contraction of the plume was determined from a shadow photograph made with the model rotated through 90° about a vertical axis, in conditions the same as for the basic experiments. The interferograms were processed to yield the temperature field in the plume above the source. For water the relation between the refractive index n and the temperature t was taken from [10, where values of n = n(t) every 1 deg C are given. The temperature was computed by interpolation for intermediate values. Subsequent processing of the experimental results was performed as in [6]. The theory of laminar convection above a

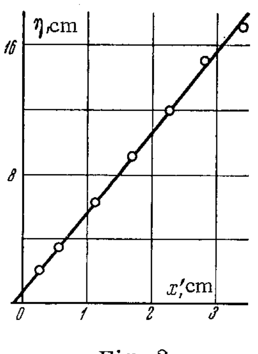

Fig. 2

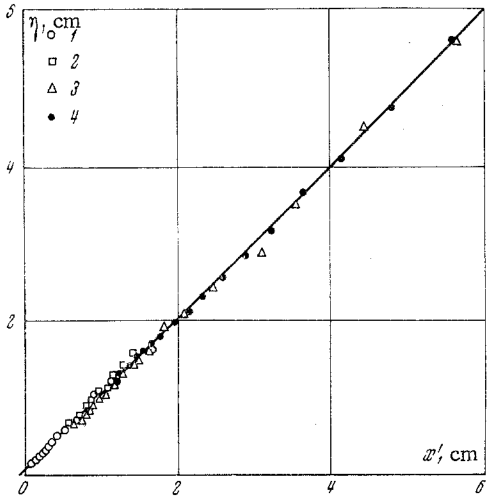

Fig. 3

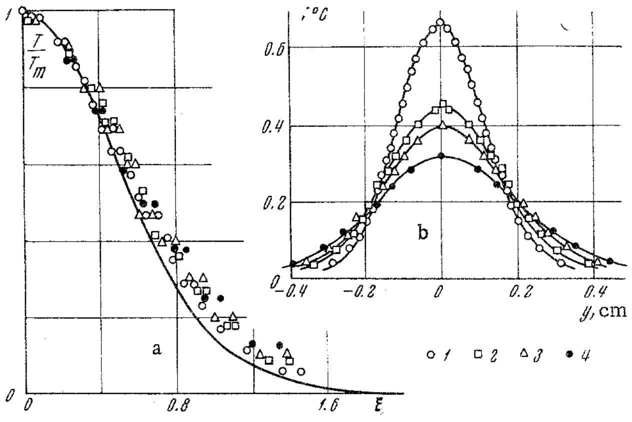

Fig. 4

linear heat source in an unbounded medium [3] gives the following dependence of the temperature in the plume T (referred to the temperature of the undisturbed fluid, assumed to be zero) on the coordinates and power of the source:

$$T = \frac{q_l^{4/5}}{B} x^{-3/5} h(\xi), \quad \xi = \left[\frac{g\beta q_l}{\rho c_p \nu^3}\right]^{1/5} x^{-2/5} y, \quad B = [g\beta\nu^2\rho^4 c_p^4]^{1/5} \tag{1.2}$$

Here $h(\xi)$ is a dimensionless function giving the temperature profile; $\xi$ is a dimensionless variable; x and y are the vertical and horizontal coordinates, calculated from the heat source; g is the acceleration due to gravity, $\beta$ is the coefficient of volume expansion; $\nu$ is the kinematic viscosity; $\rho$ is the density; $c_p$ is the specific heat at constant pressure; and $h(\xi)$ is found by solving the system of ordinary differential equations:

$$f''' + {}^3/_5\, ff'' - {}^1/_5\, (f')^2 + h = 0, \qquad h' + {}^3/_5\, P fh = 0 \tag{1.3}$$

with the boundary conditions

$$f = f'' = 0 \quad (\xi = 0), \qquad f' = h = 0 \quad (\xi = \infty) \tag{1.4}$$

It can be seen from Eq. (1.2) that in the plane of symmetry of the plume ($\xi = 0$), where the maximum temperature $T_m$ are obtained for each horizontal cross sections, the vertical coordinate x is linearly related to the quantity

$$\eta = \left[\frac{q_l^{4/5}}{BT_m}\right]^{5/3} = [h(0)]^{-5/3} x \tag{1.5}$$

This parameter is determined experimentally.

The experimental results were processed to give the graph (Fig. 2), relating $\eta$ to the vertical coordinate x', reckoned in the experiment from the center of the heated wire. The value of the compound quantity B of physical parameters of the fluid was calculated for each thermal condition and each cross section x' = const at the temperature $t = t_0 + 1/2\ T_m$, where $t_0$ is the temperature of the undisturbed fluid. Figure 2 shows this relation for air. Each point on the graph is the result of averaging experimental data for the three conditions ($q_l = 19.3 \cdot 10^{-3}$, $29.9 \cdot 10^{-3}$ and $45.3 \cdot 10^{-3}$ cal/cm sec). The fact that the experimental line does not intersect the x axis at the origin in due to the finite diameter of the wire. The coordinates x in Eq. (1.5) and x' in the experiment are related by the expression x = x' + 0.12 cm. Reference [6] obtained x = x' + 0.21 cm (the diameter of the wire used as the heater was 1 mm). The slope of the line (Fig. 2) gives the maximum value of the dimensionless temperature profile function h(0) = 0.383, which is 2.4% above the theoretical value of 0.373 from the data of [3]. In [6] the experimental value found was h(0) = 0.315, which is 15% below the theoretical value.

TABLE 1

| $x'$, см | $I, \frac{\text{cal}}{\text{cm} \cdot \text{sec}}$ | $\frac{I - q_l}{q_l}$, % | $\delta_{T_{1/4}}$ | $\delta_{g_{1/4}}$ | $k$ |
|---|---|---|---|---|---|
| 5.5 | $86.8 \cdot 10^{-3}$ | 1.4 | 0.26 | 0.59 | 1.18 |
| 6.4 | $85.7 \cdot 10^{-3}$ | 0.1 | 0.29 | 0.66 | 1.16 |
| 8.7 | $85.0 \cdot 10^{-3}$ | —0.7 | 0.33 | 0.75 | 1.18 |
| solution for $P = 7$ | | | $\xi_{T_{1/4}} = 0.83$ | $\xi_{g_{1/4}} = 2.13$ | 1.03 |

The better agreement between the experimental data of the present paper and the results of theory is due to the fact that the construction of the experimental equipment used in this work apparently provides conditions closer to those assumed in the theoretical analysis. In [6] the heated wire was located at a distance of 30.5 cm from the bottom of the insulated chamber, which had, moreover, a rectangular aperture of size 18 × 48 cm, covered with a brass mesh. This construction did not guard against inflow of cold fluid into the plume from regions located above the source level. This can also be seen from the photograph with light-scattering particles, shown in [5]. In our work the volume of fluid surrounding the heat source was bounded below by an impermeable plate, and therefore had no effect on the formation of the plume, in a thermal sense. The theory of laminar convection above a linear source [3], with which the experimental results are compared, was constructed in the boundary layer approximation, and did not take into account convective phenomena occuring below the level of the source.

2. In addition to the experiments with air, the convective plume above a heated wire was investigated in distilled water, for four thermal conditions. The experimental results with water are shown in Fig. 3; points 1, 2, 3, and 4 correspond to values of $q_l = 8.69 \cdot 10^{-3}$, $39.0 \cdot 10^{-3}$, $85.6 \cdot 10^{-3}$, $161 \cdot 10^{-3}$ cal/cm sec. The straight line drawn through the experimental points gives the relation $x = x' + 0.07$ cm, in this case. The values of h(0) found for water from the slope of the line in Fig. 3, is 1.013, with an average error of 2.5% for the individual experimental points (maximum error is 4.7%). Reference [3] did not give a solution of Eqs. (1.3) for Prandtl number $P = 7$. We carried out numerical integration of Eqs. (1.3) with boundary conditions (1.4) for $P = 7$, using a finite difference method. The value of h(0) was $0.951 \pm 1.2\%$. Thus, good agreement between experiment and theory for water was found, within the limits of experimental error.

Figure 4b shows the dimensional temperature profiles at several cross sections in the convective plume in water for the thermal conditions $q_l = 85.6 \cdot 10^{-3}$ cal/cm sec. The same figure (Fig. 4a) shows dimensionless temperature profiles in the plume, from the results of solving Eq. (1.3) with $P = 7$ (solid line) and the experimental results ($q_l = 85.6 \cdot 10^{-3}$ cal/cm sec; the experimental points 1, 2, 3, and 4 correspond to cross sections at the distances 2.9, 5.5, 6.4, and 8.7 cm from the center of the heated wire).

For one thermal condition measurements were made of the velocity profile at several sections of the plume in water. The velocities were measured by photographing light-scattering particles of aluminum powder, suspended in the liquid, with stroboscopic illumination of the flow.

Dimensionless profiles of the vertical velocity in the plume were constructed from measurements of the photographs and then the known dimensional temperature profiles in the plume at the same sections were used to compute the integral:

$$I = 2\rho c_p \int_0^\infty uT dy$$

which gives the heat flux density carried away by the plume. The results are shown in Table 1. The values of the integral I for the various sections agree, within an accuracy no worse than 1.4%, with the thermal power dissipated by the source ($q_l = 85.6 \cdot 10^{-3}$ cal/cm sec).

Table 1 also gives results of comparing the thicknesses of the thermal and hydrodynamic boundary layers for a plume in water, from the experimental data and the results of numerical solution of Eqs. (1.3). The quantity $\delta_T$ (or $\delta_g$) with subscript 1/4 is the distance horizontally from the plume plane of symmetry, to the point at which the temperature or the vertical velocity at the given section has fallen off by a factor of 4 with respect to the maximum value. It can be seen from Table 1 that these results are also in agreement. The ratio between the thicknesses of the thermal and hydrodynamic boundary layers obtained in this paper agrees with the results of the approximate solution:

$$k = \frac{\delta_T}{\delta_g} \sqrt{P}$$

where the constant k, which is independent of the value of P, is close to 1.1 for the plume.

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